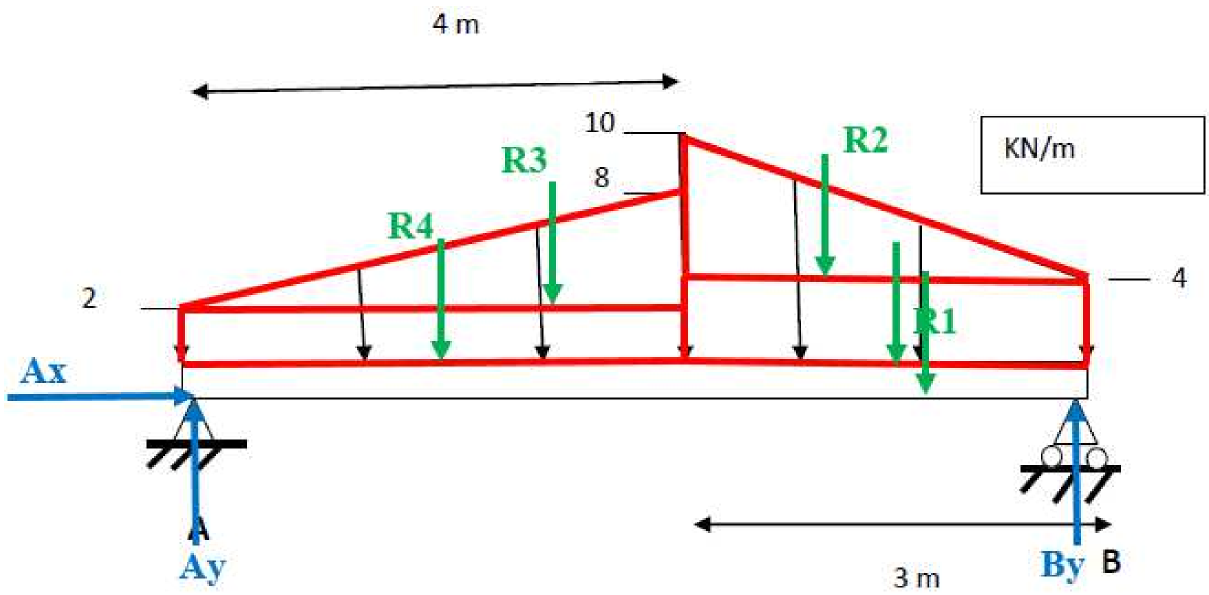
Contrôle 2013

Exercice 1 :



Calculons d'abord les résultantes :

Calcul de R1 :

$$R1 = 4 \times 3 = 12\ kN$$

Point d'application : $x1 = 4 + 3/2 = 7{,}5\ m\ de\ point\ A$

Calcul de R2 :

$$R2 = (10 - 4) \times 3 \times \frac{1}{2} = 9\ kN$$

Point d'application : $x2 = 4 + \frac{3}{3} = 5\ m\ de\ point\ A$

Calcul de R3 :

$$R3 = (8 - 2) \times 4 \times \frac{1}{2} = 12\ kN$$

Point d'application : $x3 = 2 \times \frac{4}{3} = \frac{8}{3}\ m\ de\ point\ A$

Calcul de R4 :

$$R4 = 2 \times 4 = 8\ kN$$

Point d'application : $x4 = 2\ m\ de\ point\ A$

Les réactions :

$$\sum M_A = 0 \quad \Leftrightarrow By \times 7 - R1 \times 7{,}5 - R2 \times 5 - R3 \times \frac{8}{3} - R4 \times 2$$

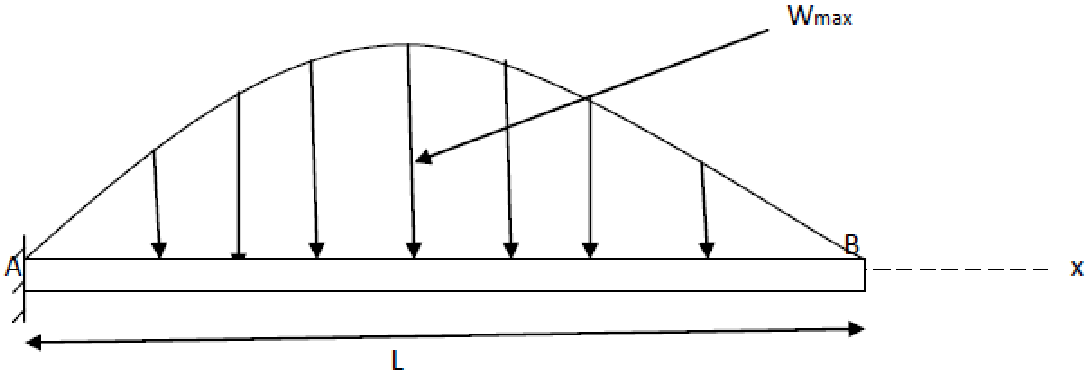
$$\Rightarrow By = 26{,}14\ kN$$

$$\sum F_y = 0 \quad \Leftrightarrow Ay + By - R1 - R2 - R3 - R4 = 0$$

$$\Rightarrow Ay = 14{,}86\ kN$$

Et $Ax = 0$

b/



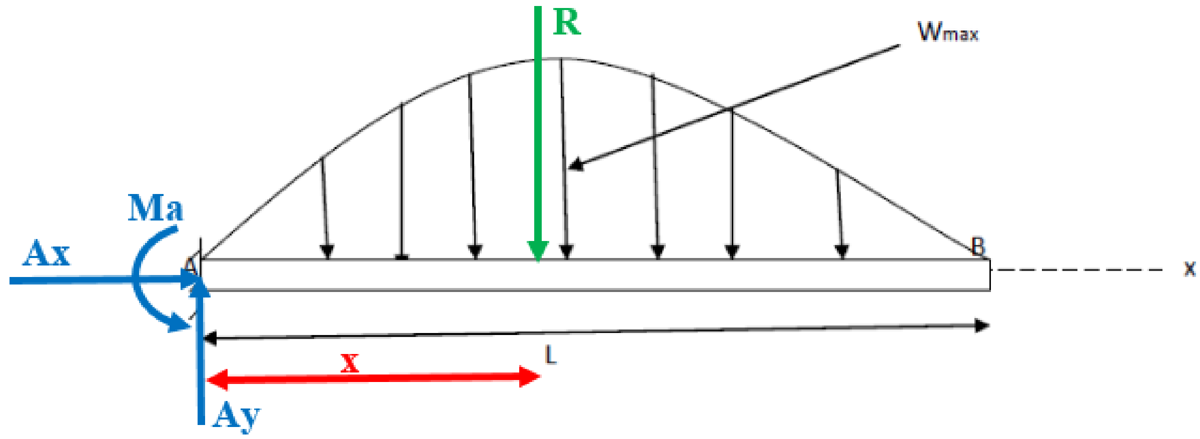
W = W_{max} sin(π x/L)

Calcul de la résultante :

$$R = \int_0^L W dx = \int_0^L W_{max} \sin(\pi \frac{x}{L}) dx$$

$R = int\left(Wmax \cdot \sin\left(\frac{Pi \cdot x}{L} \right), x = 0 .. L \right);$

$$R = \frac{2\, Wmax\, L}{\pi}$$

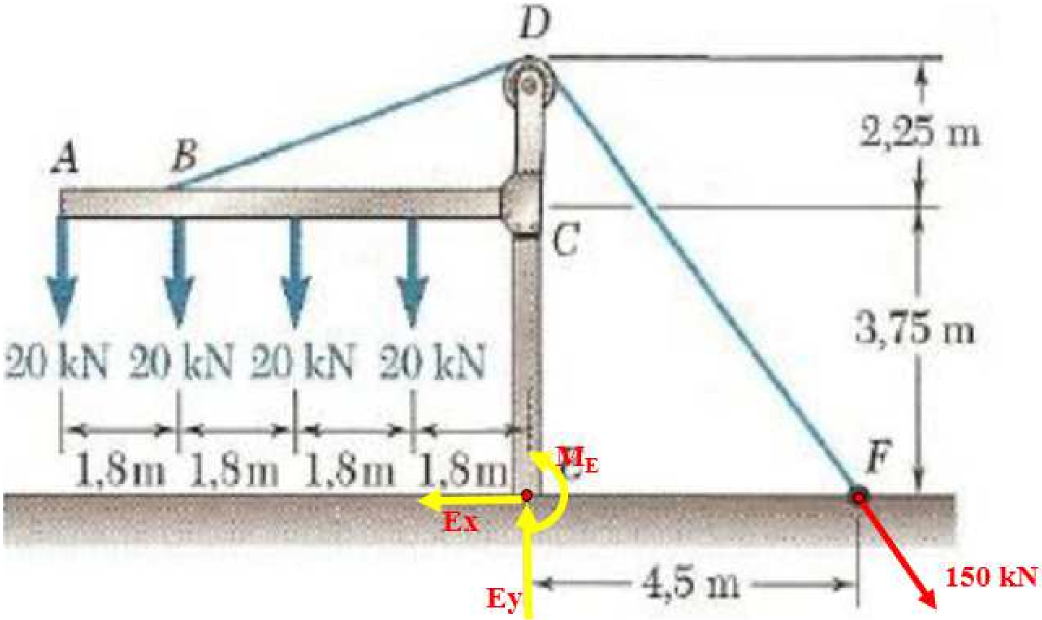


$$\sum Fy = 0 \Leftrightarrow Ay - R = 0$$

$$\Rightarrow Ay = \frac{2LW_{max}}{\pi}$$

$$Ma = R \times x = \frac{2LW_{max}}{\pi} \times \frac{L}{2} = \frac{L^2 W_{max}}{\pi}$$

Exercice 2 :



Cherchons tout d'abord la distance DF :

$$DF = \sqrt{4,5^2 + 6^2} = 7,5\ m$$

Les réactions :

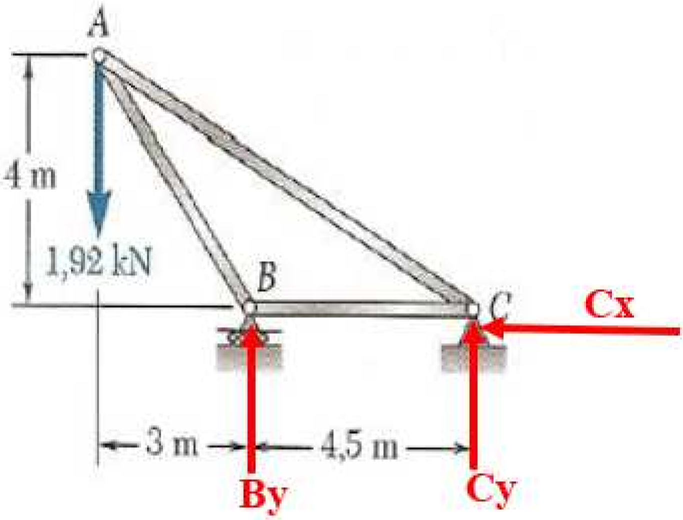
$$\sum F_x = 0 \Leftrightarrow \left(150 \times \frac{4,5}{7,5}\right) - E_x = 0 \ \Rightarrow \ E_x = 90\ kN$$

$$\sum F_y = 0 \Leftrightarrow E_y - \left(150 \times \frac{6}{7,5}\right) - (4 \times 20) = 0 \ \Rightarrow \ E_y = 200\ kN$$

$$\sum M_E = 0 \Leftrightarrow M_E - \left(150 \times \frac{6}{7,5} \times 4,5\right) + (20 \times 1,8) + (20 \times 3,6) + (20 \times 5,4) + (20 \times 7,2) = 0 \ \Rightarrow \ M_E = 180\ kN.m$$

Exercice 3 :

DCL global :



$$\sum M_C = 0 \Leftrightarrow 1{,}92 \times (3 + 4{,}5) - By \times 4{,}5 = 0$$

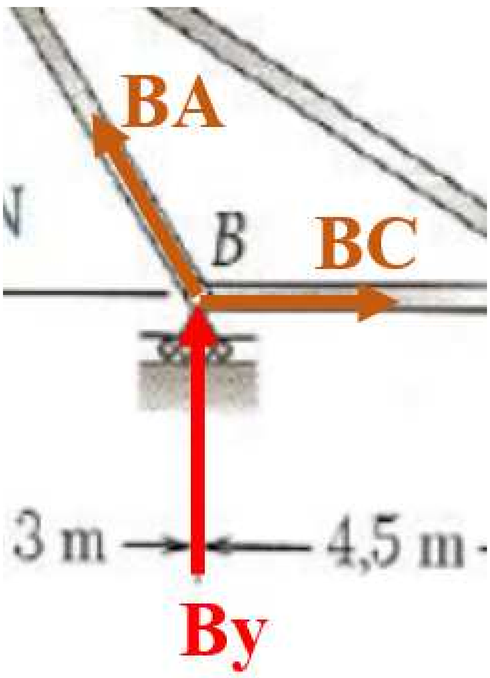
$$\Rightarrow By = 3{,}2\ kN$$

$$\sum F_y = 0 \Leftrightarrow By + Cy - 1{,}92 = 0$$

$$\Rightarrow Cy = -1{,}28\ kN$$

$$\sum F_x = 0 \Leftrightarrow Cx = 0$$

Nœud B :



$$(1): \sum Fx = 0 \Leftrightarrow BC - BA \times cos\alpha = 0$$

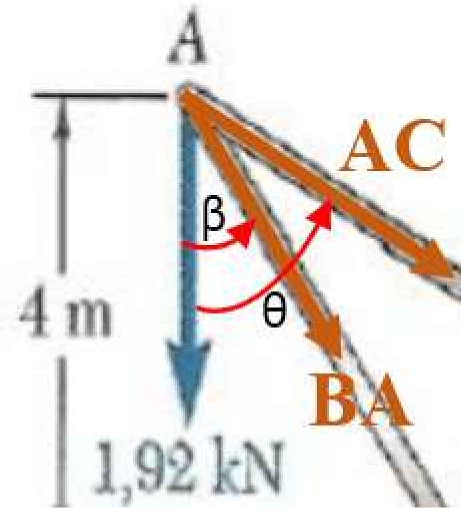
$$(2): \sum Fy = 0 \Leftrightarrow By + BA \times sin\alpha = 0$$

$$tan\alpha = \frac{4}{3} \Rightarrow \alpha = 53.13$$

$$(2) \Rightarrow : BA = \frac{-By}{sin\alpha} = -4\ kN$$

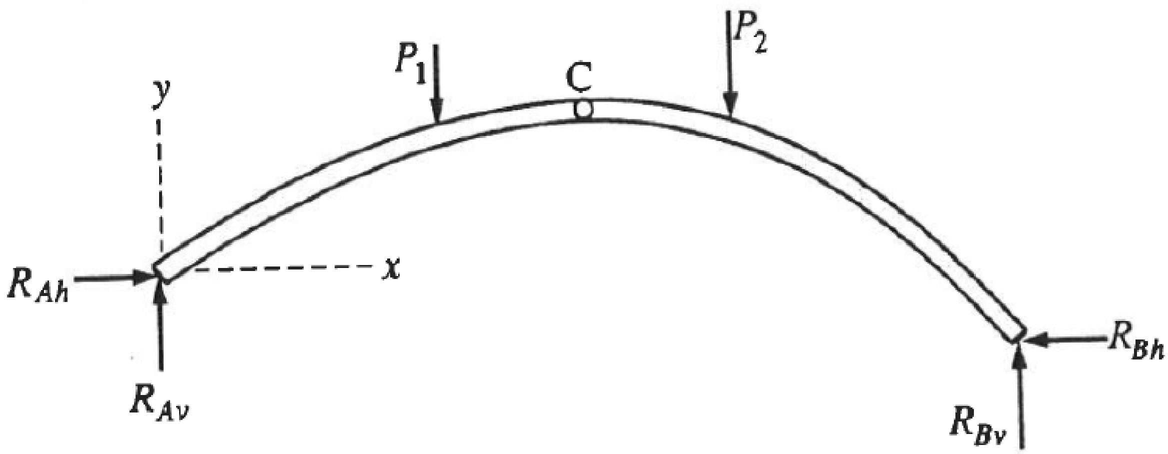
$$(1) \Rightarrow BC = BA \times cos\alpha = -2{,}4\ kN$$

Nœud A :



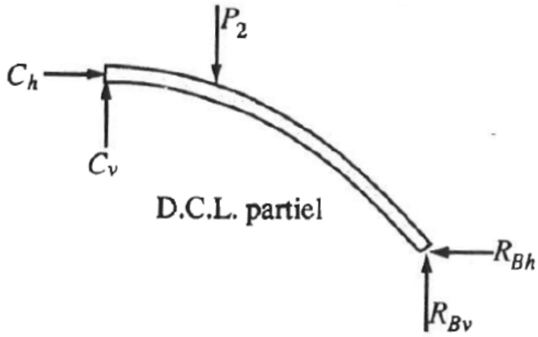
$$(3): \sum Fx = 0$$
$$\Leftrightarrow AC \times sin\theta + BA \times sin\beta = 0$$
$$tan\beta = \frac{3}{4} \Rightarrow \beta = 36{,}86$$
$$tan\theta = \frac{7{,}5}{4} \Rightarrow \theta = 61.92$$
$$(3) \Rightarrow AC = \frac{-BA \times sin\beta}{sin\theta} = 2{,}72\ kN$$

b/ DCL global :



$$(1): \sum Fx = 0 \Leftrightarrow R_{Ah} - R_{Bh} = 0$$
$$(2): \sum Fy = 0 \Leftrightarrow R_{Av} + R_{Bv} - P_1 - P_2 = 0$$
$$(3): \sum M_A = 0 \Leftrightarrow -R_{Bh} \times h + R_{Bv} \times L - P_1 \times a_1 - P_2 \times a_2 = 0$$

Il faut chercher une quatrième équation, donc on fait un DCL local sur la partie CB et on obtient :


P2
Ch
Cv
D.C.L. partiel
RBh
RBv

$$(4): \sum M_C = 0 \Leftrightarrow -R_{Bh} \times (h + h') + R_{Bv} \times \frac{L}{2} - P_2 \times (a_2 - a_C) = 0$$

```
restart;
equ1 := Rah - Rbh = 0;
equ2 := Rav + Rbv - P1 - P2 = 0;
equ3 := Rbv·L - Rbh·h - P1·a1 - P2·a2 = 0;
equ4 := Rbv·l/2 - Rbh·(h + H) - P2·(a2 - ac) = 0;
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$$equ1 := Rah - Rbh = 0$$

$$equ2 := Rav + Rbv - P1 - P2 = 0$$

$$equ3 := Rbv\,L - P1\,a1 - P2\,a2 - Rbh\,h = 0$$

$$equ4 := \frac{Rbv\,l}{2} - Rbh\,(h + H) - P2\,(a2 - ac) = 0$$

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solve({equ4, equ3}, {Rbv, Rbh});
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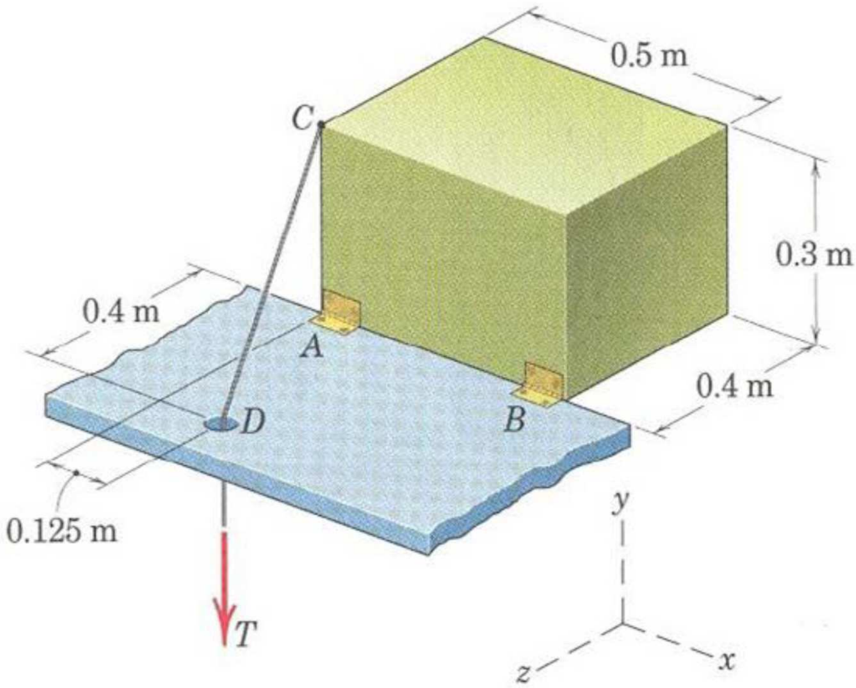
$$\left\{Rbh = -\frac{2\,L\,P2\,a2 - 2\,L\,P2\,ac - P1\,a1\,l - P2\,a2\,l}{2\,HL + 2\,L\,h - hl}, Rbv = \frac{2\,(HP1\,a1 + HP2\,a2 + P1\,a1\,h + P2\,ac\,h)}{2\,HL + 2\,L\,h - hl}\right\}$$

Cherchons les réactions au point A :

$$Rah = -\frac{2\,L\,P2\,a2 - 2\,L\,P2\,ac - P1\,a1\,l - P2\,a2\,l}{2\,HL + 2\,L\,h - hl}$$

$$Rav = \frac{2\,HL\,P1 + 2\,HL\,P2 - 2\,HP1\,a1 - 2\,HP2\,a2 + 2\,L\,P1\,h + 2\,L\,P2\,h - 2\,P1\,a1\,h - P1\,hl - 2\,P2\,ac\,h - P2\,hl}{2\,HL + 2\,L\,h - hl}$$

Exercice 4 :



$$C = \begin{pmatrix} 0 \\ 0{,}3 \\ 0 \end{pmatrix} \;\; et \;\; D = \begin{pmatrix} 0{,}125 \\ 0 \\ 0{,}4 \end{pmatrix} \;\; donc \;\; \overrightarrow{CD} = 0{,}125\,\vec{\imath} - 0{,}3\,\vec{\jmath} + 0{,}4\,\vec{k}$$

$$et \;\; CD = \sqrt{0{,}125^2 + 0{,}3^2 + 0{,}4^2} = 0{,}515$$

$$\vec{T} = T\overrightarrow{\lambda_{CD}} = T\frac{\overrightarrow{CD}}{CD}$$

$$\Rightarrow \vec{T} = T\left(\frac{0{,}125\,\vec{\imath} - 0{,}3\,\vec{\jmath} + 0{,}4\,\vec{k}}{0{,}515}\right) = (0{,}242 \times T)\,\vec{\imath} - (0{,}582 \times T)\vec{\jmath} + (0{,}776 \times T)\vec{k}$$

Afin de trouver la tension, on calcule la somme des moment suivant l’axe Ox :

$$\sum M_{Ox} = 0 \Leftrightarrow 0{,}3 \times Tz - 0{,}2 \times W = 0 \quad avec\; Tz = 0{,}776 \times T$$

$$\Rightarrow T = \frac{0{,}2 \times W}{0{,}3 \times 0{,}776} = \frac{0{,}2 \times 200 \times 9{,}81}{0{,}3 \times 0{,}776} = 1685{,}56\; N$$