TD4 - MMC

Exercice 1 :

1/ Déterminons σ_{ij} :

$$\sigma_{ij} = \begin{bmatrix} 10 & -6 & 0 \\ -6 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2/ La partie hydrostatique :

$$\sigma_m = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

La partie déviatoire :

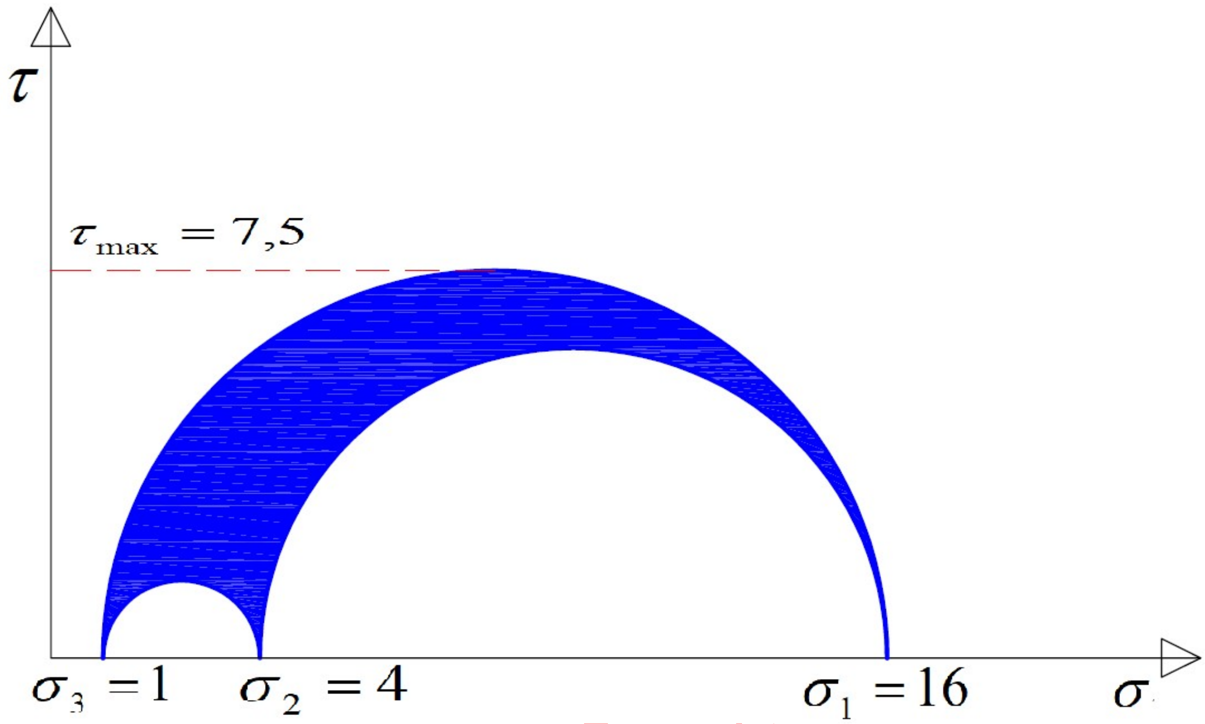
$$S_{ij} = \begin{bmatrix} 3 & -6 & 0 \\ -6 & 3 & 0 \\ 0 & 0 & -6 \end{bmatrix}$$

3/ déterminons les contraintes principales :

$$\det(\sigma_{ij} - \lambda I) = \det\begin{bmatrix} 10-\lambda & -6 & 0 \\ -6 & 10-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix} = -(\lambda-16)(\lambda-4)(\lambda-1)$$

$$\Rightarrow \begin{cases} \sigma_1 = \lambda_1 = 16 \\ \sigma_2 = \lambda_2 = 4 \\ \sigma_3 = \lambda_3 = 1 \end{cases}$$

4/ Le cercle de Mohr :



Exercice 2 :

1/ Calcul des contraintes principales :

$$\sigma = \begin{bmatrix} 0,7\alpha & 3,6\alpha & 0 \\ 3,6\alpha & 0,7\alpha & 0 \\ 0 & 0 & 7,6 \end{bmatrix}$$

$$\det(\sigma - \lambda I)\det\begin{bmatrix} 0,7\alpha-\lambda & 3,6\alpha & 0 \\ 3,6\alpha & 0,7\alpha-\lambda & 10 \\ 0 & 10 & 7,6-\lambda \end{bmatrix} = 0$$

$$\begin{cases} \sigma_1 = \lambda_1 = 7,6 \\ \sigma_2 = \lambda_2 = 5,5\alpha \\ \sigma_3 = \lambda_3 = -2\alpha \end{cases}$$

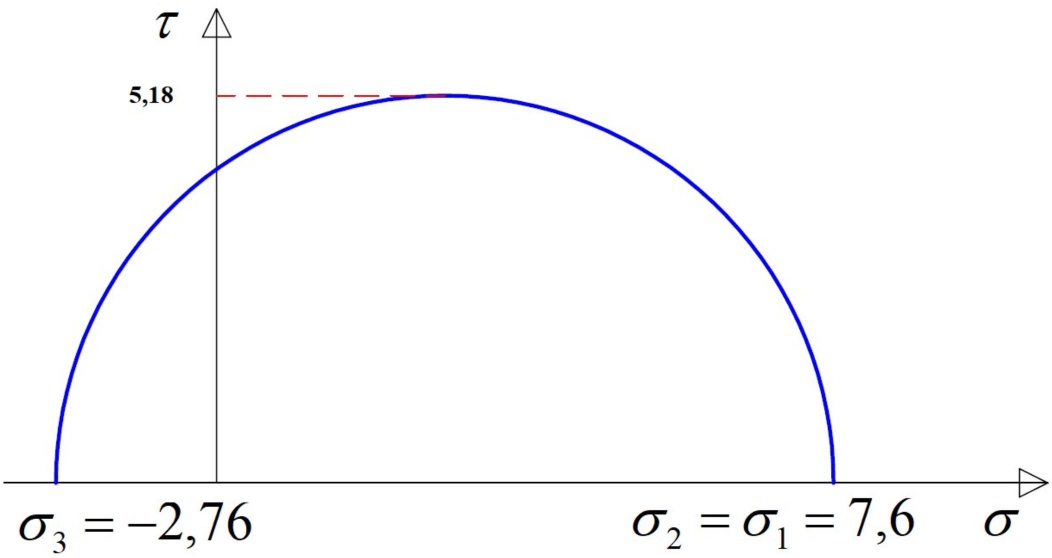
Dans la base principale, les torseurs de contraIntes, s'écrivent :

$$\sigma = \begin{bmatrix} 7,6 & 0 & 0 \\ 0 & 5,5\alpha & 0 \\ 0 & 0 & -2\alpha \end{bmatrix}$$

2/

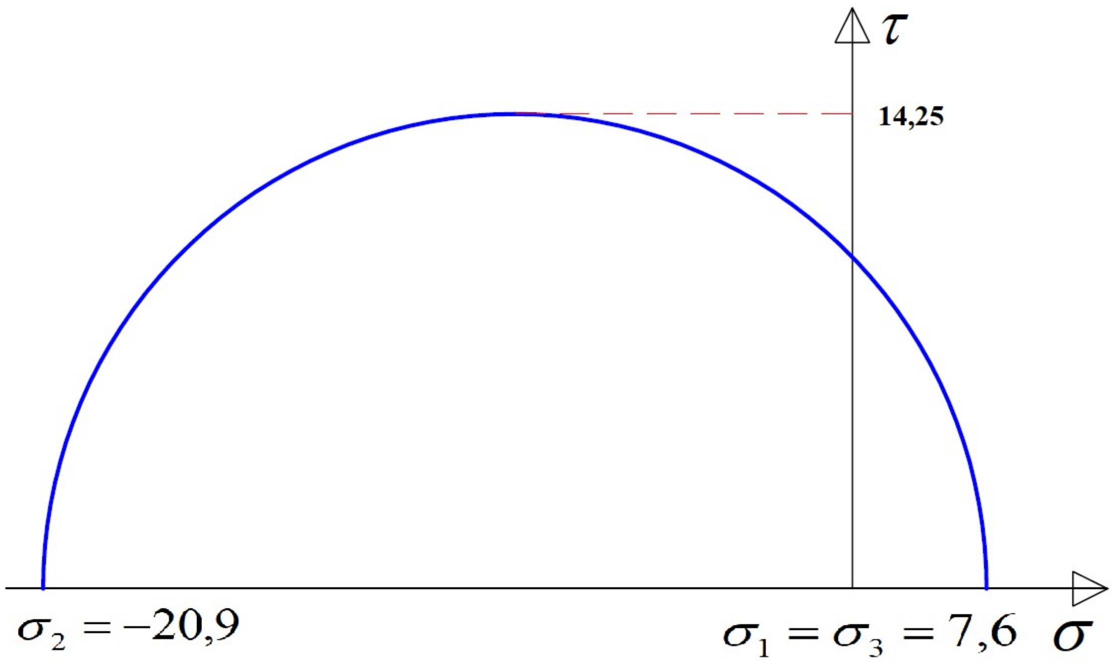
- Dans le cas ou $\sigma_1 = \sigma_2$:

$$\Rightarrow \alpha = \frac{7,6}{5,5} = 1,38$$



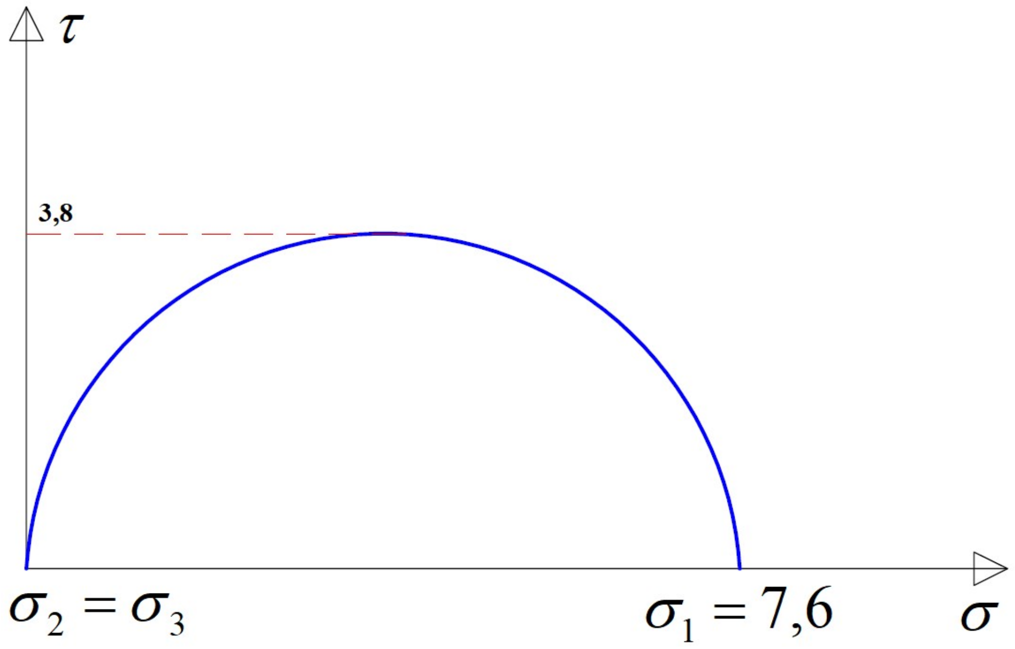
- Dans le cas ou $\sigma_1 = \sigma_3$:

$$\Rightarrow \alpha = -3,8$$



- Dans le cas ou $\sigma_2 = \sigma_3$:

$\Rightarrow \alpha = 0$



3/ Posons $\alpha = 1$:

a/ Déterminons les vecteurs propres :

$$\sigma = \begin{bmatrix} 7,6 & 0 & 0 \\ 0 & 5,5 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$Pour\ \sigma_1 = \lambda_1 = 7,6\ MPa$$

$$(\sigma - \sigma_1 I)\vec{V}_1 = \vec{0} \quad \Rightarrow \quad \vec{V}_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

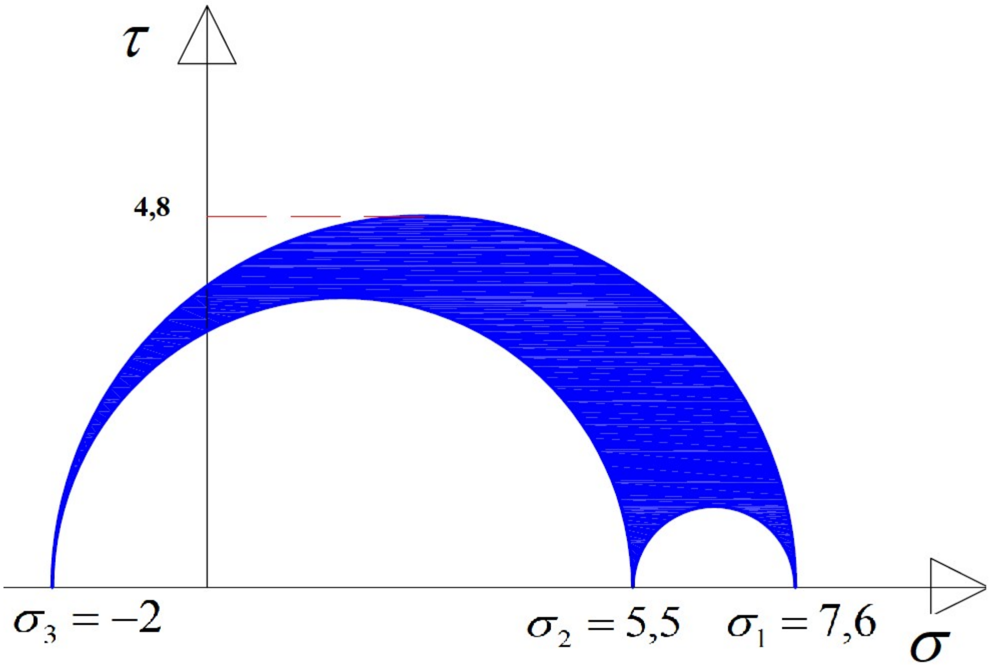
$$Pour\ \sigma_2 = \lambda_2 = 5,5\ MPa$$

$$(\sigma - \sigma_2 I)\vec{V}_2 = \vec{0} \quad \Rightarrow \quad \vec{V}_2 = \begin{pmatrix} 0,6 \\ 0,8 \\ 0 \end{pmatrix}$$

$$Pour\ \sigma_3 = \lambda_3 = -2$$

$$(\sigma - \sigma_3 I)\vec{V}_3 = \vec{0} \quad \Rightarrow \quad \vec{V}_3 = \begin{pmatrix} 0,8 \\ -0,6 \\ 0 \end{pmatrix}$$

b/ Les cercles de Mohr :



c/ Calculons la contraintes pour la direction :

$$\vec{T}(M,\vec{n}) = \sigma.\vec{n}$$

$$\{n\} = \begin{Bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{Bmatrix}$$

$$\Rightarrow \vec{T}(M,\vec{n}) = \begin{bmatrix} 0,7 & 3,6 & 0 \\ 3,6 & 2,8 & 0 \\ 0 & 0 & 7,6 \end{bmatrix} . \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix} = \begin{bmatrix} 2,4 \\ 4,51 \\ 0 \end{bmatrix}$$

d/ La valeur de la contrainte de cisaillement maximum :

$$\tau = \sqrt{T^2 - (\sigma_m)^2}$$

$$\sigma_m = \vec{n}.\vec{T}(M,\vec{n})$$

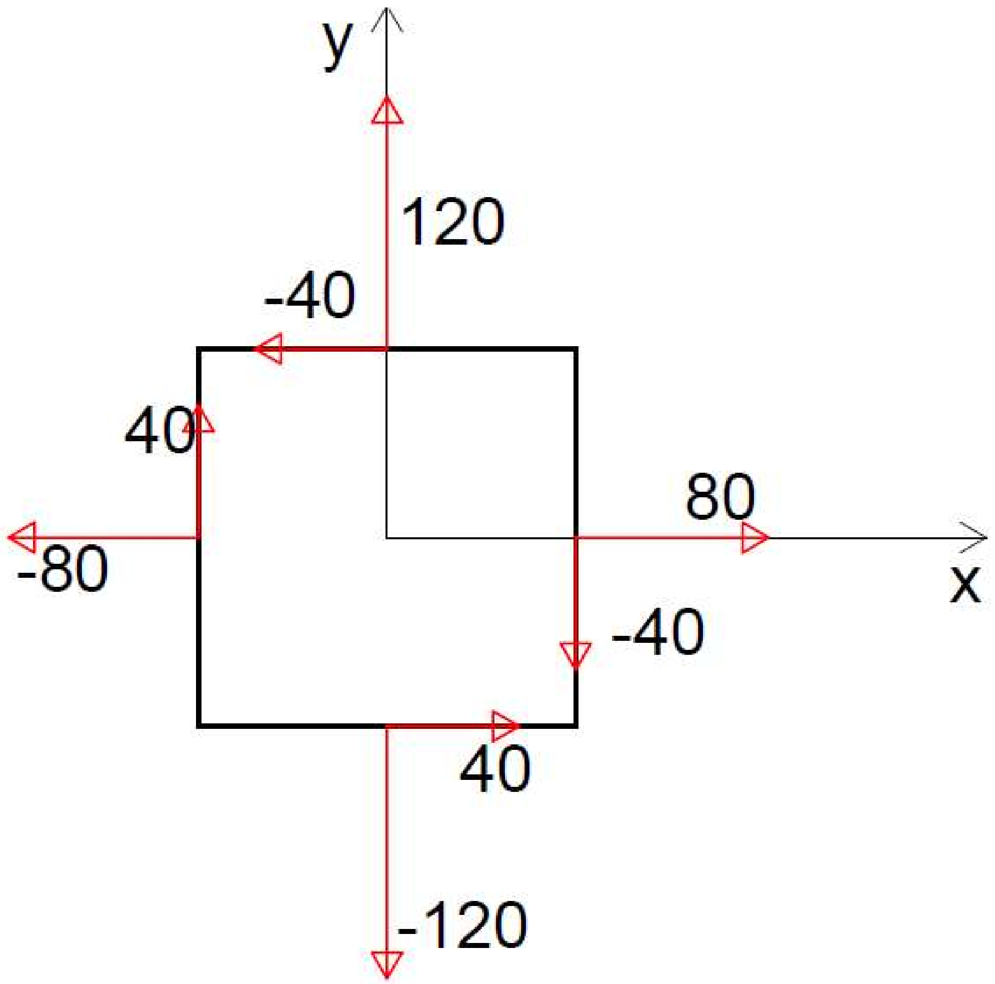
$$\Rightarrow \sigma_m = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \end{pmatrix} \times \begin{pmatrix} 2{,}4 \\ 4{,}51 \\ 0 \end{pmatrix}$$

$$\sigma_m = 4{,}3\ MPa$$

$$\Rightarrow \tau = 5{,}10\ MPa$$

Exercice 4 :

1/



2/ Les contraintes principales :

$$\sigma = \begin{bmatrix} 80 & -40 & 0 \\ -40 & 120 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

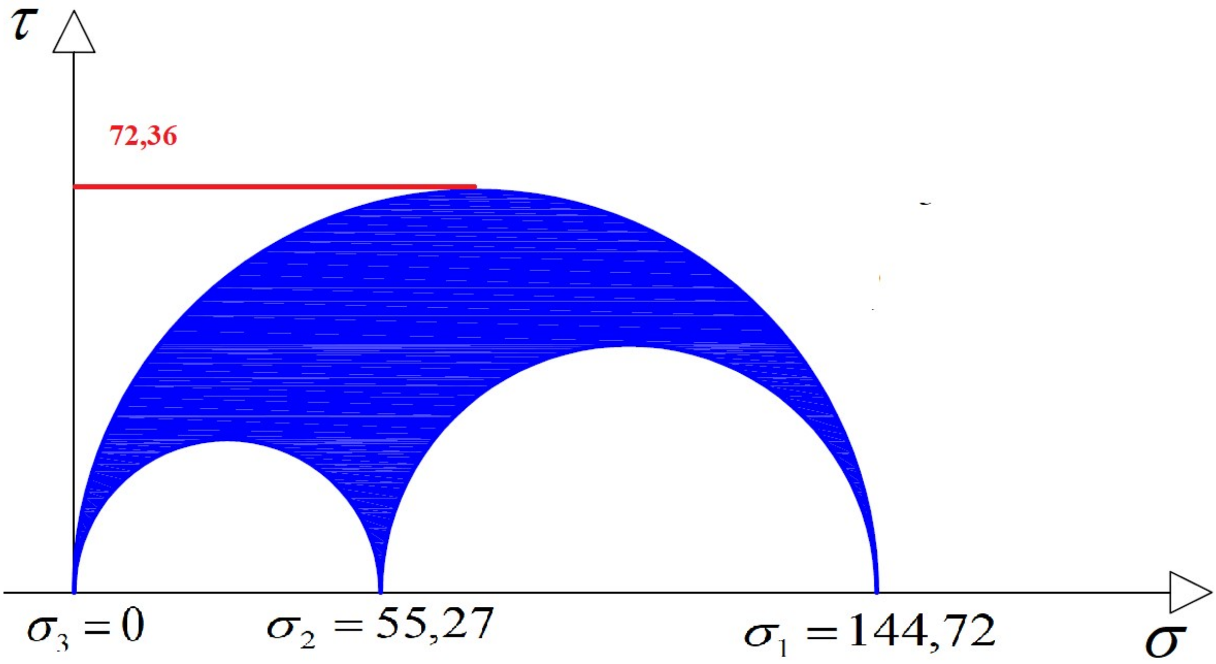
$$\det(\sigma - \lambda I) = \det \begin{bmatrix} 80-\lambda & -40 & 0 \\ -40 & 120-\lambda & 0 \\ 0 & 0 & -\lambda \end{bmatrix} = 0$$

$$\begin{cases} \sigma_1 = \lambda_1 = 144{,}72\ MPa \\ \sigma_2 = \lambda_2 = 55{,}27\ MPa \\ \sigma_3 = \lambda_3 = 0 \end{cases}$$

Dans la base principale, les torseurs de contraIntes, s'écrivent :

$$\sigma = \begin{bmatrix} 144{,}27 & 0 & 0 \\ 0 & 55{,}27 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Le cercle de Mohr :



Les directions principales :

$$Pour\,\sigma_1 = \lambda_1 = 144{,}72\,MPa$$

$$(\sigma - \sigma_1 I)\vec{V}_1 = \vec{0} \quad \Rightarrow \quad \vec{V}_1 = \begin{pmatrix} 0{,}52 \\ -0{,}85 \\ 0 \end{pmatrix}$$

$$Pour\,\sigma_2 = \lambda_2 = 55{,}27\,MPa$$

$$(\sigma - \sigma_2 I)\vec{V}_2 = \vec{0} \quad \Rightarrow \quad \vec{V}_2 = \begin{pmatrix} 0{,}85 \\ 0{,}52 \\ 0 \end{pmatrix}$$

$$Pour\,\sigma_3 = \lambda_3 = 0$$

$$\vec{V}_3 = \vec{V}_1 \wedge \vec{V}_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

4/ La contrainte de Von mises :

$$\sigma_{VM} = \sqrt{\frac{1}{2}\left((\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_3 - \sigma_2)^2\right)}$$

$$\Rightarrow \sigma_{VM} = 126{,}49\,MPa$$

La contrainte de Tresca :

$$\sigma_T = 2\tau_{\max} = \max(\sigma_1, \sigma_2, \sigma_3) - \min(\sigma_1, \sigma_2, \sigma_3)$$

$$\Rightarrow \sigma_T = 144{,}72\,MPa$$