TD1 -*MMC*

Exercice 1 :

1/

F(2x) = 8x+5

$F(x^2) = 4x^2 +5$

F(x+4) = 4x + 17

2/

$$X + Y = \begin{pmatrix} 5 \\ 7 \\ 9 \end{pmatrix}$$

$$X.Y = 32$$

$$\|X\| = \sqrt{14}$$

$$3X = \begin{pmatrix} 3 \\ 6 \\ 9 \end{pmatrix}$$

3/

$$3A = \begin{pmatrix} 3 & 0 & 0 \\ 3 & 6 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$

$$Z = A.X = \begin{pmatrix} 1 & 5 & 9 \end{pmatrix}$$

$$C = A.B = \begin{pmatrix} 3 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\det(A) = 6$$

$$Tr(A) = 6$$

Exercice 3 :

1/ tenseur gradient F :

$$\overline{\overline{F}} = \begin{pmatrix} \cos(\omega t) & -\sin(\omega t) & 0 \\ \sin(\omega t) & \cos(\omega t) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Tenseur des dilatations :

$$\overline{\overline{C}} = {}^t\overline{\overline{F}}.\overline{\overline{F}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Tenseur des déformations :

$$\overline{\overline{E}} = \frac{1}{2}\left(\overline{\overline{C}} - 1\right) = 0_{3,3}$$

2/ Mouvement rigidifiant (mouvement d'un solide rigide).

3/ Dilatation :

$$\lambda(e_1) = \sqrt{C_{11}} = 1$$
$$\lambda(e_2) = \sqrt{C_{22}} = 1$$
$$\lambda(e_3) = \sqrt{C_{33}} = 1$$

4/ Glissement :

$$\gamma_{12} = Arc\sin\left(\frac{C_{12}}{\sqrt{C_{11}}\sqrt{C_{22}}}\right) = 0$$

$$\gamma_{13} = Arc\sin\left(\frac{C_{13}}{\sqrt{C_{11}}\sqrt{C_{33}}}\right) = 0$$

$$\gamma_{23} = Arc\sin\left(\frac{C_{23}}{\sqrt{C_{22}}\sqrt{C_{33}}}\right) = 0$$

5/ le Jacobien de la transformation :

$$J = \frac{dV}{dV_0} = \det \overline{\overline{F}} = 1$$

$$\text{à chaque instant } t : \ \rho_0 = \rho_1 = \rho_2 = \ldots$$

6/ le champ de vitesse en coordonnées lagrangiennes :

$$V_1 = \frac{\partial x_1}{\partial t} = -X_1 \omega \sin(\omega t) - X_2 \omega \cos(\omega t)$$

$$V_2 = \frac{\partial x_2}{\partial t} = X_1 \omega \cos(\omega t) - X_2 \omega \sin(\omega t)$$

$$V_3 = \frac{\partial x_3}{\partial t} = 0$$

Le champ d'accélération en coordonnées lagrangiennes :

$$\gamma_1 = \frac{\partial V_1}{\partial t} = -X_1 \omega^2 \cos(\omega t) + X_2 \omega^2 \sin(\omega t)$$

$$\gamma_2 = \frac{\partial V_2}{\partial t} = X_1 \omega^2 \sin(\omega t) - X_2 \omega^2 \cos(\omega t)$$

$$\gamma_3 = \frac{\partial V_3}{\partial t} = 0$$

7/ les coordonnées initiales à partir des coordonnées actuelles :

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \cos(\omega t) & -\sin(\omega t) \\ \sin(\omega t) & \cos(\omega t) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{bmatrix} = \frac{1}{\det}{}^t\big(com(M)\big)$$

$$\Rightarrow \begin{bmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$d'ou : \quad X_1 = \cos(wt)x_1 + \sin(wt)x_2$$

$$X_2 = -\sin(wt)x_1 + \cos(wt)x_2$$

$$on\ en\ déduit\ V_1\ et\ V_2 :$$

$$\begin{cases} V_1 = \dfrac{\partial X_1}{\partial t} = -\omega \sin(\omega t)x_1 + \omega \cos(\omega t)x_2 \\ V_2 = \dfrac{\partial X_2}{\partial t} = -\omega \cos(\omega t)x_1 - \omega \sin(\omega t)x_2 \\ V_3 = \dfrac{\partial X_3}{\partial t} = 0 \end{cases}$$

$$Calcul\ de\ \gamma_1, \gamma_2\ et\ \gamma_3$$

$$\begin{cases} \gamma_1 = \dfrac{\partial V_1}{\partial t} = -\omega^2 \sin(\omega t)x_1 - \omega^2 \cos(\omega t)x_2 \\ \gamma_2 = \dfrac{\partial V_2}{\partial t} = \omega^2 \sin(\omega t)x_1 - \omega^2 \cos(\omega t)x_2 \\ \gamma_3 = \dfrac{\partial V_3}{\partial t} = 0 \end{cases}$$