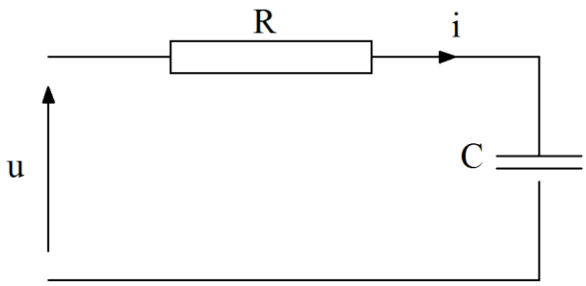
TD 1 Automatique linéaire

Au tour de la fonction de transfert

Exercice 1 :



$$E_C = \frac{1}{2} m\dot{y}^2(t)$$

$$E_p = \frac{1}{2} K y^2(t)$$

$$\Rightarrow L = E_C - E_p = \frac{1}{2} m\dot{y}^2(t) - \frac{1}{2} K y^2(t)$$

D'après Lagrange-Euler :

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} = -\frac{\partial D}{\partial \dot{y}} + u(t)$$

$$\Rightarrow m\ddot{y}(t) + ky(t) = -\alpha\dot{y}(t) + u(t)$$

$$\Rightarrow m\ddot{y}(t) + \alpha\dot{y}(t) + ky(t) = u(t)$$

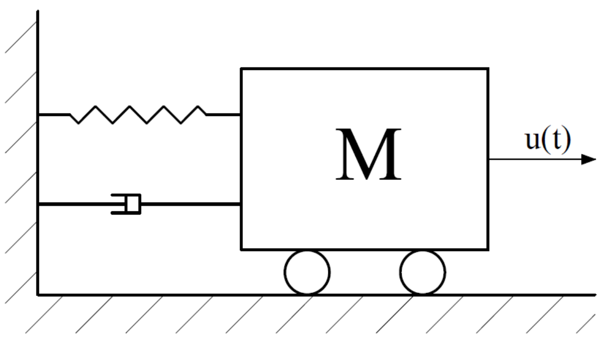
TL =>

$$mp^2Y(p) + \alpha pY(p) + KY(p) = u(p)$$

$$\Rightarrow Y(p)(mp^2 + \alpha p + K) = u(p)$$

$$\Rightarrow H(p) = \frac{Y(p)}{u(p)} = \frac{1}{mp^2 + \alpha p + K}$$

Exercice 2 :



$$\sum Fext = m\vec{a}$$

On projette sue l'axe Ox :

$$-Ky(t) - \alpha\dot{y}(t) + u(t) = m\ddot{y}(t)$$

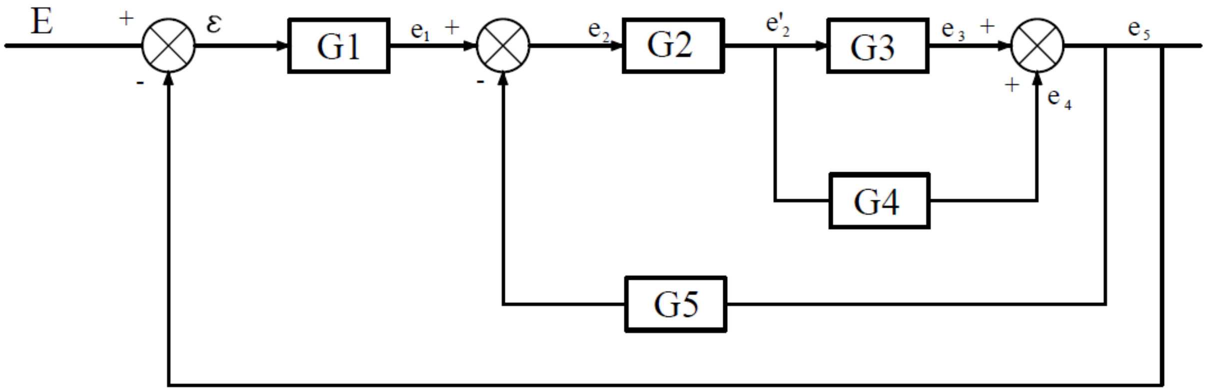
Transformé de Laplace :

$$\Longrightarrow -KY(p) - \alpha pY(p) + u(p) = mp^2Y(p)$$

$$\Longrightarrow Y(p).(mp^2 + \alpha p + K) = u(P)$$

$$\Longrightarrow H(p) = \frac{Y(p)}{u(p)} = \frac{1}{mp^2 + \alpha p + K}$$

Exercice 3 :



$$\varepsilon = E - e_5$$

$$e_1 = \varepsilon.G1$$

$$e_2 = e_1 - e_5.G5$$

$$e_2' = e_2.G2$$

$$e_4 = e_2'.G4$$

$$e_3 = e_2'.G3$$

$$e_5 = e_3 + e_4$$

$$H = \frac{e_5}{E}$$

$$e_5 = e_3 + e_4 = e_2'.G3 + e_2'.G4 = e_2' \times (G3 + G4)$$

$$\Longrightarrow e_5 = e_2.G2 \times (G3 + G4) = e_2 \times (G2.G3 + G2.G4)$$

$$\Longrightarrow e_5 = (e_1 - e_5.G5) \times (G2.G3 + G2.G4)$$

$$\Longrightarrow e_5 \times \big(1 + G5 \times (G2.G3 + G2.G4)\big) = e_1 \times (G2.G3 + G2.G4)$$

$$\Rightarrow e_5 \times \left(1 + G5 \times (G2.G3 + G2.G4)\right) = \varepsilon \times (G1.G2.G3 + G1.G2.G4)$$

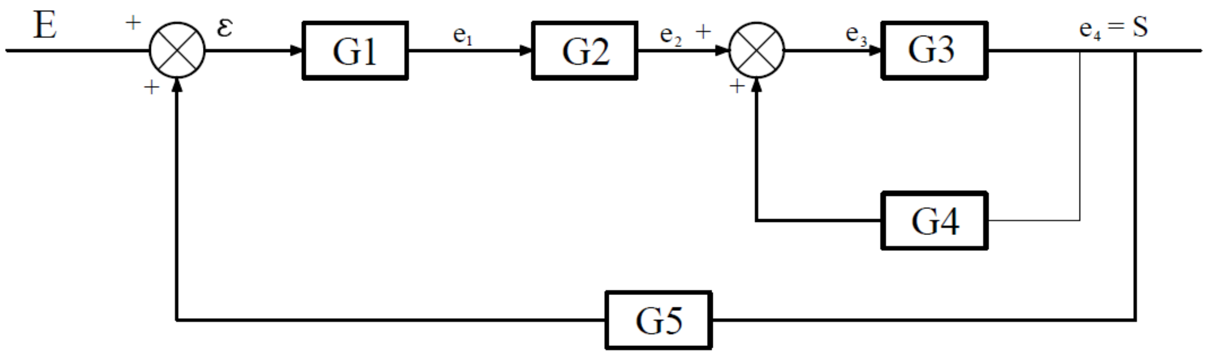
$$\Rightarrow e_5 \times \left(1 + G5 \times (G2.G3 + G2.G4)\right) = (E - e_5) \times (G1.G2.G3 + G1.G2.G4)$$

$$\Rightarrow e_5 \times \left(1 + G5.G2.(G3 + G4) + G1.G2.(G3 + G4)\right) = E \times G1.G2 \times (G3 + G4)$$

$$\Rightarrow e_5 = \frac{E \times G1 \times G2 \times (G3 + G4)}{1 + G5 \times G2 \times (G3 + G4) + G1 \times G2 \times (G3 + G4)}$$

$$\Rightarrow H = \frac{G1 \times G2 \times (G3 + G4)}{1 + G5 \times G2 \times (G3 + G4) + G1 \times G2 \times (G3 + G4)}$$

Exercice 4 :



$$\varepsilon = E + e_4.G5$$

$$e_1 = \varepsilon.G1$$

$$e_2 = e_1.G1$$

$$e_3 = e_2 + e_4.G4$$

$$e_4 = e_3.G3$$

$$H = \frac{e_4}{E}$$

$$e_4 = e_3.G3 = (e_2 + e_4.G4) \times G3$$

$$\Rightarrow e_4(1 - G3G4) = e_2.G3$$

$$\Rightarrow e_4(1 - G3G4) = e_1.G2G3 = \varepsilon.G1.G2.G3$$

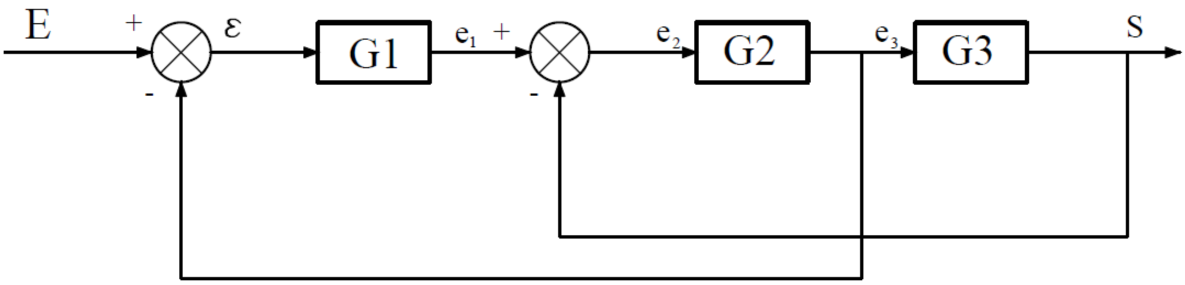
$$\Rightarrow e_4(1 - G3G4) = (E + e_4.G5).G1.G2.G3$$

$$\Rightarrow e_4 = \frac{E.G1.G2.G3}{1 - G3.G4 - G1.G2.G3.G5}$$

$$\Rightarrow H = \frac{G1.G2.G3}{1 - G3.G4 - G1.G2.G3.G5}$$

Exercice 5 :

Système 1 :



$$S = e_3 . G3$$
$$\varepsilon = E - e_3$$
$$e_3 = e_2 . G2$$
$$e_2 = e_1 - S$$
$$e_1 = \varepsilon . G1$$

On a :

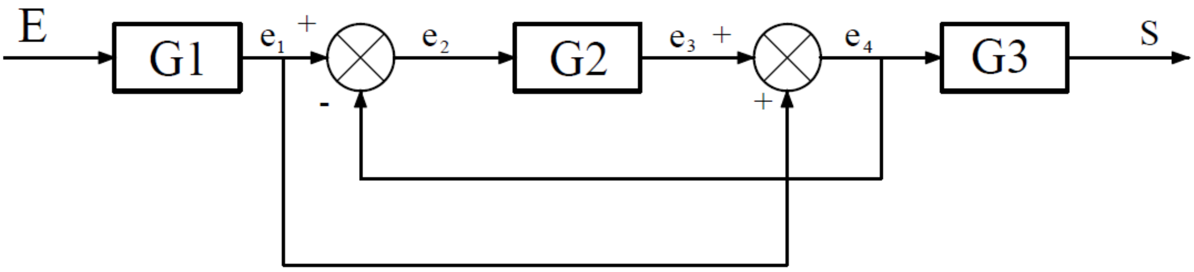
$$S = e_3 . G3 = e_2 . G2 . G3$$
$$\Longrightarrow S = e_1 . G2 . G3 - S . G2 . G3$$
$$\Longrightarrow S(1 + G2 . G3) = \varepsilon . G1 . G2 . G3$$
$$\Longrightarrow S(1 + G2 . G3) = (E - e_3) G1 . G2 . G3$$
$$\Longrightarrow S(1 + G2 . G3) = E . G1 . G2 . G3 - e_3 . G1 . G2 . G3$$
$$\Longrightarrow S(1 + G2 . G3) = E . G1 . G2 . G3 - \left(\frac{S}{G3}\right) . G1 . G2 . G3$$

$$\Longrightarrow S = \frac{E . G1 . G2 . G3}{1 + G2 . G3 + G1 . G2}$$

Donc :

$$\Longrightarrow H = \frac{G1 . G2 . G3}{1 + G2 . G3 + G1 . G2}$$

Système 2 :



$$S = e_4.G3$$

$$e_4 = e_1 + e_3$$

$$e_3 = e_2.G2$$

$$e_2 = e_1 - e_4$$

$$e_1 = E.G1$$

On a :

$$S = e_4.G3 = (e_1 + e_3) \times G3 = (E.G1 + e_2.G2) \times G3$$

$$\Rightarrow S = E.G1.G3 + e_2.G2.G3$$

$$\Rightarrow S = E.G1.G3 + (e_1 - e_4).G2.G3$$

$$\Rightarrow S = E.G1.G3 + e_1.G2.G3 - e_4.G2.G3$$

$$\Rightarrow S = E.G1.G3 + E.G1.G2.G3 - \left(\frac{S}{G3}\right).G2.G3$$

$$\Rightarrow S = \frac{E.(G1.G3 + G1.G2.G3)}{1 + G2}$$

Donc :

$$\Rightarrow H = \frac{G1.G3 + G1.G2.G3}{1 + G2}$$

$$E_C = \frac{1}{2}m\dot{y}^2(t)$$

$$E_p = \frac{1}{2}Ky^2(t)$$

$$\Rightarrow L = E_C - E_p = \frac{1}{2}m\dot{y}^2(t) - \frac{1}{2}Ky^2(t)$$

D'après Lagrange-Euler :

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} = -\frac{\partial D}{\partial \dot{y}} + u(t)$$

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$$\Longrightarrow \; Y(p)(mp^2 + \alpha p + K) = u(p)$$

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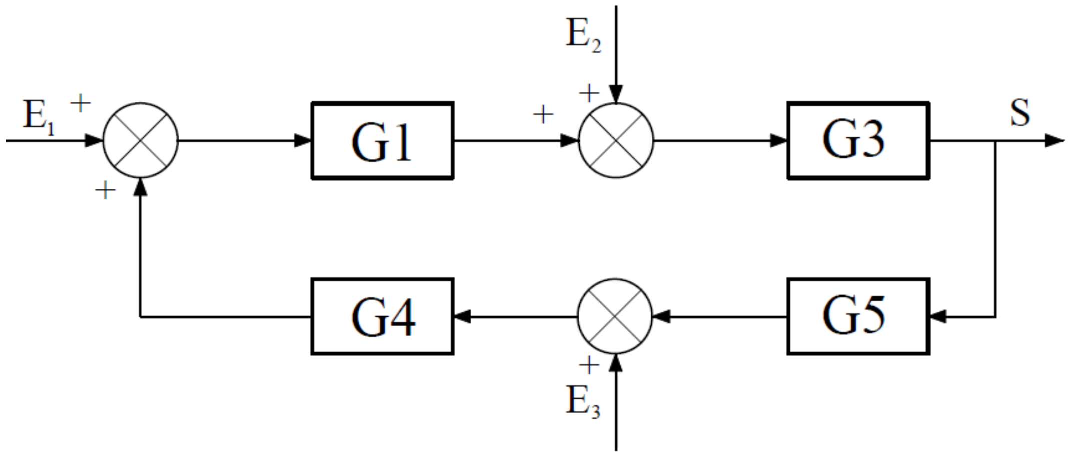
Exercice 6 :



En appliquant la technique de superposition :

$$E_2 = 0 \; et \; E_3 = 0 \quad \Longrightarrow \; H_1 = \frac{S}{E_1} = \frac{G1G3}{1 - G1G3G4G5}$$

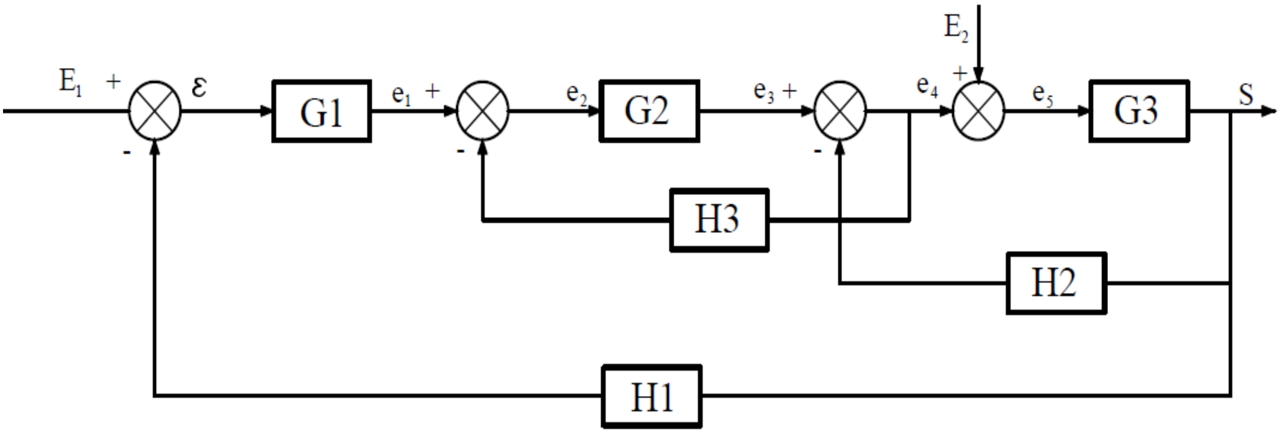
$$E_1 = 0 \; et \; E_3 = 0 \quad \Longrightarrow \; H_2 = \frac{S}{E_2} = \frac{G3}{1 - G1G3G4G5}$$

$$E_1 = 0 \; et \; E_1 = 0 \quad \Longrightarrow \; H_3 = \frac{S}{E_3} = \frac{G5}{1 - G1G3G4G5}$$

$$\Longrightarrow H_{total} = H_1 + H_2 + H_3$$

$$\Longrightarrow H_{total} = \frac{G3 + G1G3 + G5}{1 - G1G3G4G5}$$

Exercice 7 :



En appliquant la technique de superposition :

$$E_2 = 0 \implies H_1(p) = \frac{S}{E_1}$$

$$S = e_5.G3$$

$$\varepsilon = E1 - H1.S$$

$$e_1 = \varepsilon.G1$$

$$e_2 = e_1 - H2S.$$

$$e_3 = e_2.G2$$

$$e_4 = e_3 - H2.S$$

$$e_5 = e_4$$

$$on\ a:\ S = e_5.G3 = (e_3 - H2.S)G3 = (e_2.G2 - H2.S).G3$$

$$\implies S = (e_1.G2 - e_4.G2.H3 - H2.S).G3$$

$$\implies S = (e_1.G2 - (e_3 - H2.S).G2.H3 - H2.S).G3$$

$$\implies S = (E1.G1.G2 - H1.S.G2.G1 - (e_2.G2 - H2.S).G2.H3 - H2.S).G3$$

$$\implies S = (E1.G1.G2 - H1.S.G2.G1 - (e_3 - H2.S).G2.H3 - H2.S).G3$$

Or : $e_4 = e_3 - H2.S = e_5 \implies e_4 = \frac{S}{G3}$

$$\implies e_3 = S.H2 + \frac{S}{G3}$$

Donc :

$$H_1(p) = \frac{S}{E_1} = \frac{G1.G2.G3}{1 + H1.G1.G2.G3 + H3.G2 + H2.G3}$$

$$E_1 = 0 \implies H_2(p) = \frac{S}{E_2}$$

$$e_1 = S.H1.G1$$

$$e_2 = e_1 - H3.e_4$$

$$e_3 = e_2.G2$$

$$e_4 = e_3 - H2.S$$

$$e_5 = e_4 + E2$$

$$S = e_5.G3 = e_4.G3 + E2.G3 = e_3.G3 - S.H2.G3 + E2.G3$$

$$\implies S = e_2.G2.G3 - S.H2.G3 + E2.G3$$

$$\implies S = e_1.G2.G3 - e_4.H3.G2.G3 - S.H2.G3 + E2.G3$$

$$\implies S = S.H1.G1.G2.G3 - \left(\frac{S - E2.G3}{G3}\right).H3.G2.G3 - S.H2.G3 + E2.G3$$

$$\implies S = S.H1.G1.G2.G3 - (S - E2.G3).H3.G2 - S.H2.G3 + E2.G3$$

$$\implies S = \frac{E2.H3.G2.G3 + E2.G3}{1 - H1.G1.G2.G3 + H3.G2}$$

$$\implies H_2(p) = \frac{S}{E_2} = \frac{H3.G2.G3 + G3}{1 - H1.G1.G2.G3 + H3.G2}$$

Donc :

$$\implies H_{total} = H_1(p) + H_2(p)$$

$$\implies H_{total} = \frac{G1.G2.G3}{1 + H1.G1.G2.G3 + H3.G2 + H2.G3} + \frac{H3.G2.G3 + G3}{1 - H1.G1.G2.G3 + H3.G2}$$